

Neural·Pragmatic

Natural

Language

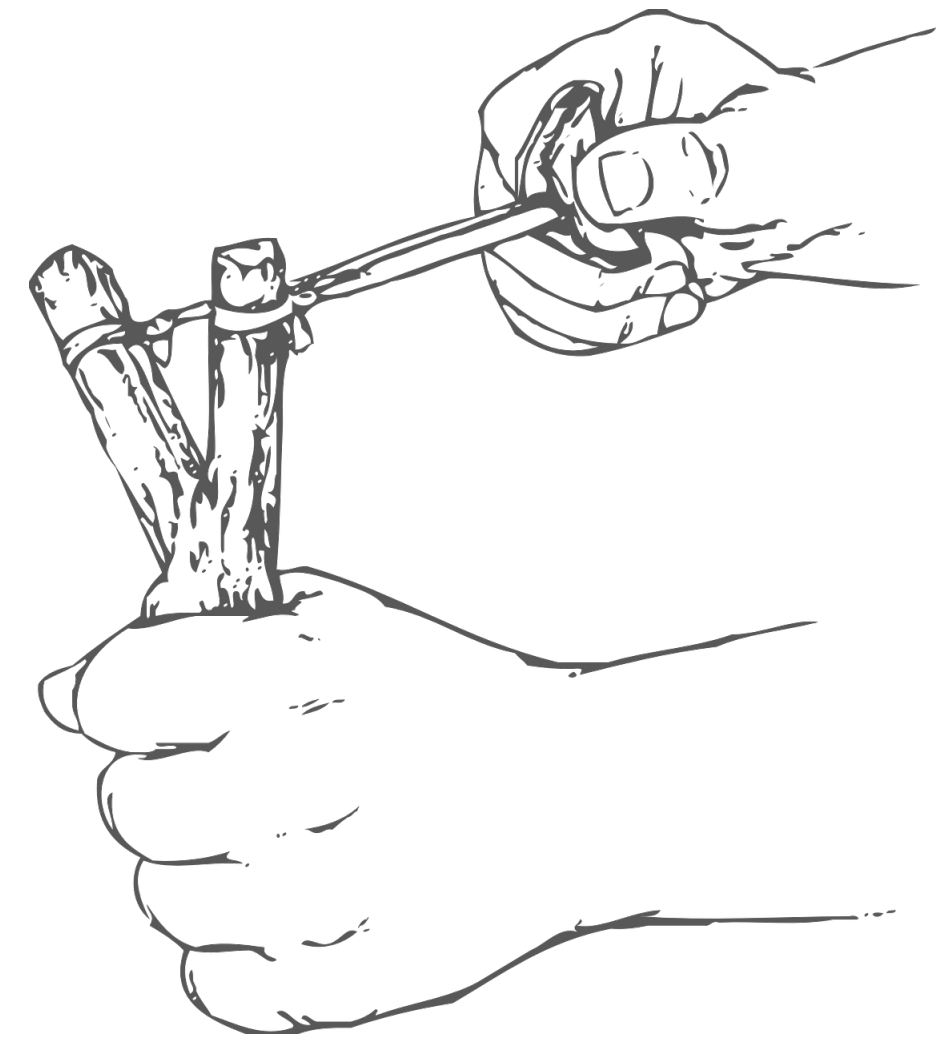
Generation

N·P

NLG

Learning goals

1. Become familiar with ANNs:
 - a. mathematical notation in matrix-vector form
 - b. weights & biases (slopes & intercepts), score, activation function, hidden layers, prediction
2. Be able to use PyTorch to implement a feed-forward ANN:
 - a. building the model by hand
 - b. using built-in helper functions (`nn.Module`, `DataLoader` ...)



Units (neurons)

- ▶ input vector:

$$\mathbf{x} = [x_1, \dots, x_n]^T$$

- ▶ weight vector:

$$\mathbf{w} = [w_1, \dots, w_n]^T$$

- ▶ bias:

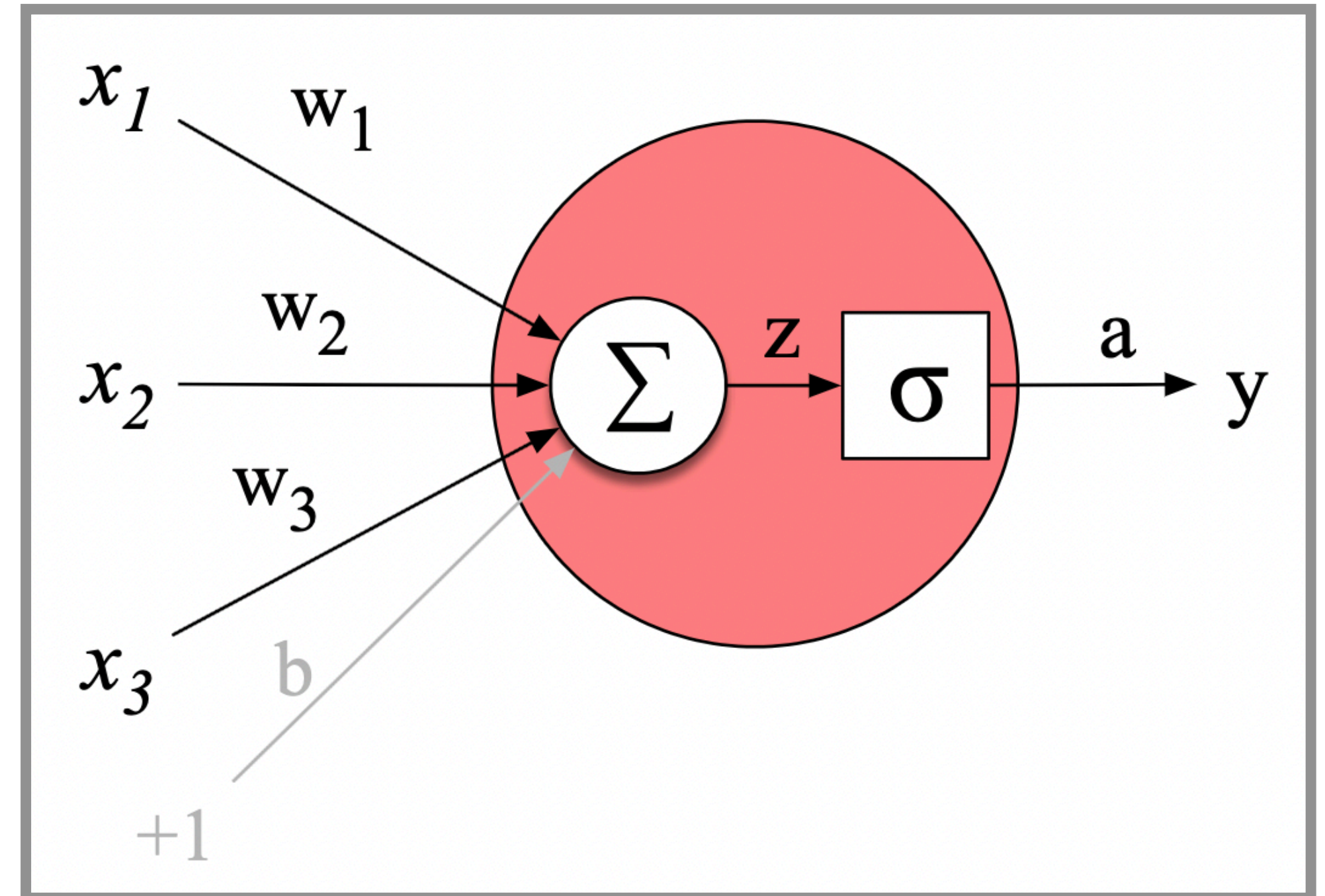
$$b$$

- ▶ score:

$$z = b + \sum_{j=1}^n w_j x_j = b + \mathbf{w} \cdot \mathbf{x}$$

- ▶ activation level:

$$a = f(z), \text{ where } f \text{ is the } \mathbf{activation\ function}$$



Common activation functions

- ▶ perceptron:

$$f(z) = \delta_{z>0}$$

- ▶ sigmoid:

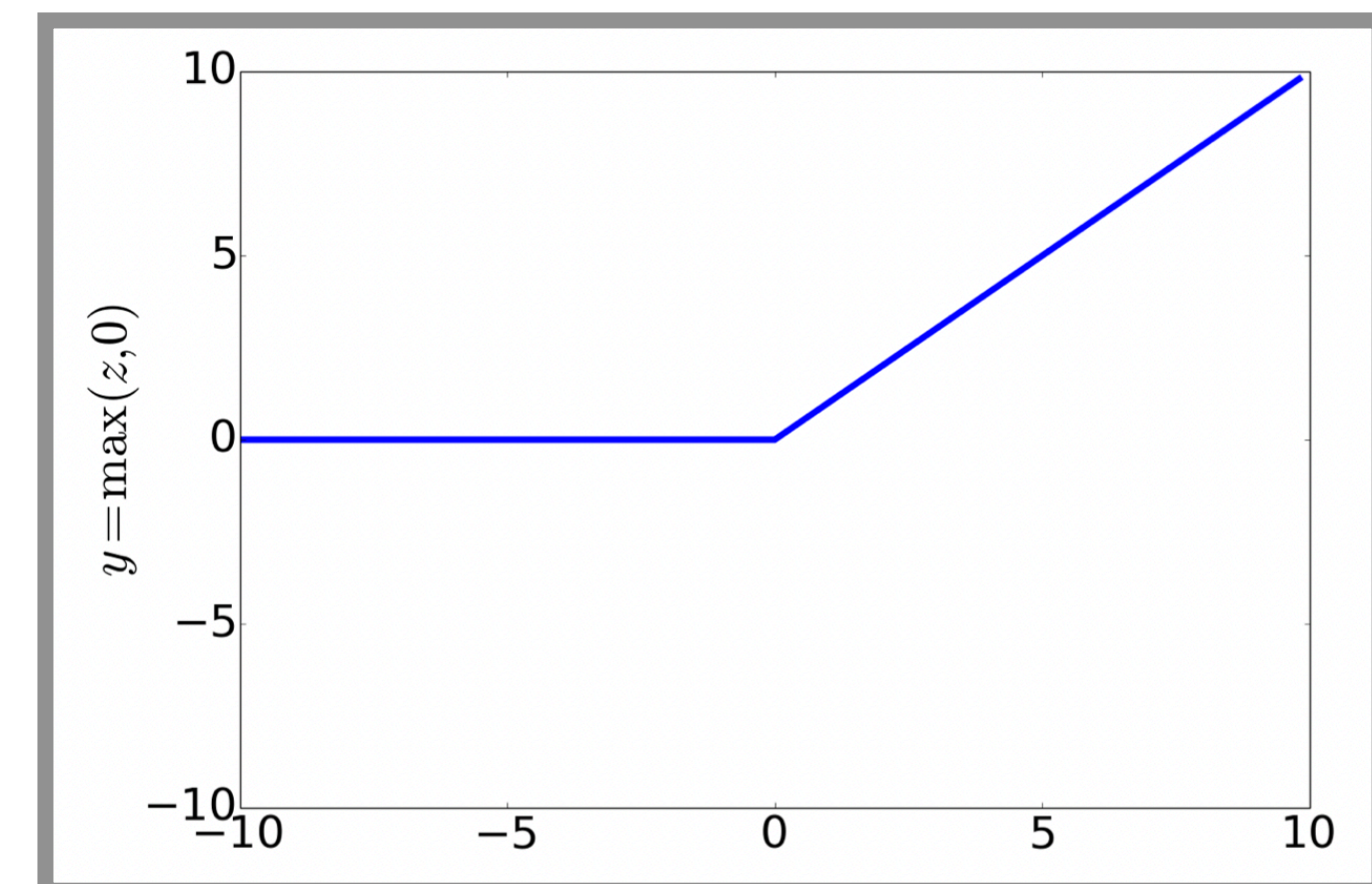
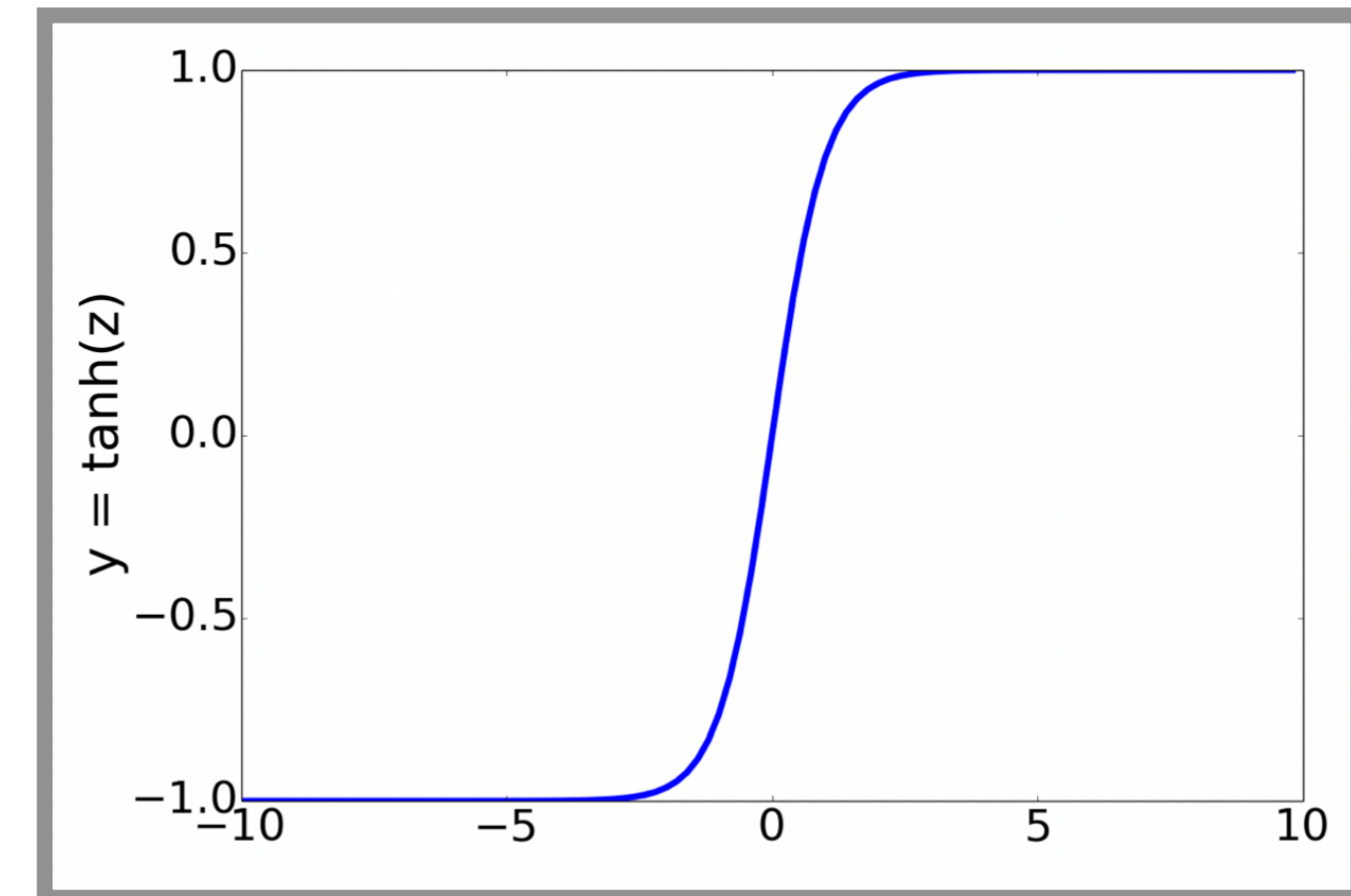
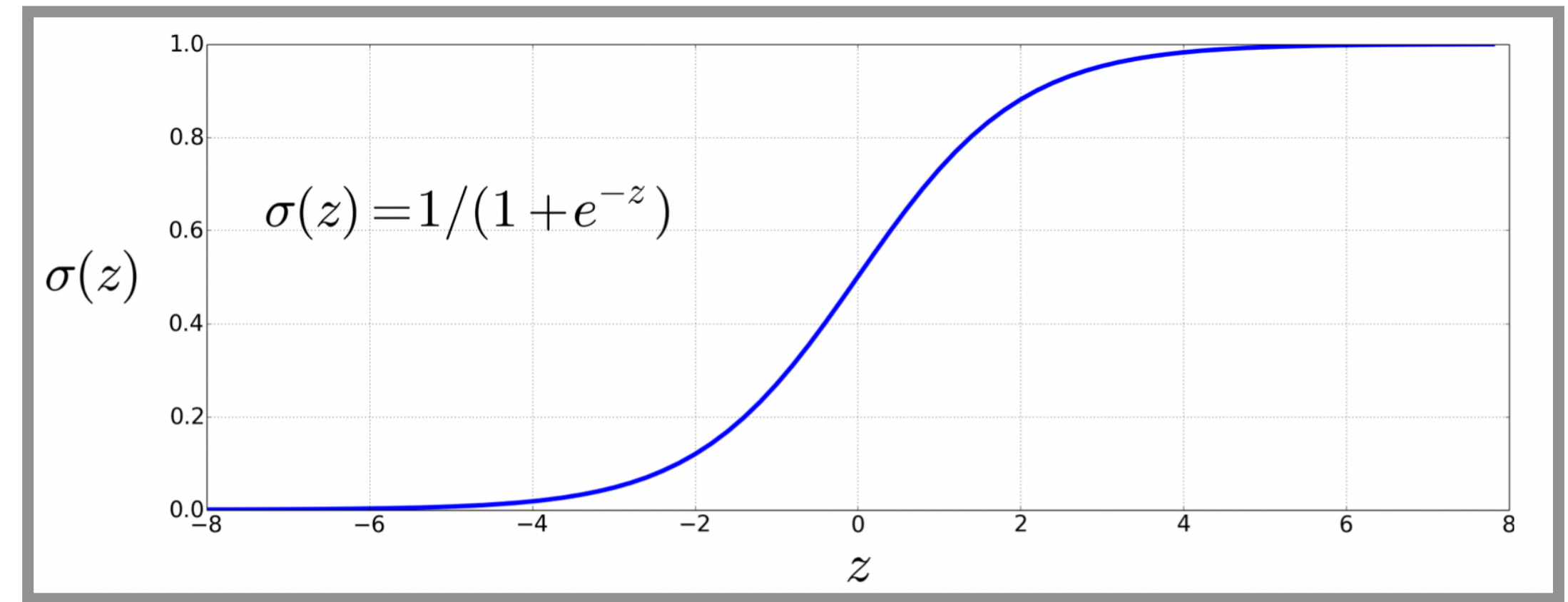
$$f(z) = \sigma(z) = \frac{1}{1 + \exp(-z)}$$

- ▶ hyperbolic tangent:

$$f(z) = \tanh(z) = \frac{\exp(z) - \exp(-z)}{\exp(z) + \exp(-z)}$$

- ▶ rectified linear unit:

$$f(z) = \text{ReLU}(z) = \max(z, 0)$$



Recap: Matrix Multiplication

$$c_{11} = a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} + a_{14}b_{41}$$
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \\ b_{41} & b_{42} & b_{43} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}$$

2×4 4×3 2×3

$$c_{22} = a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} + a_{24}b_{42}$$
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \\ b_{41} & b_{42} & b_{43} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}$$

Feedforward neural network (one hidden layer)

- ▶ input:

$$\mathbf{x} = [x_1, \dots, x_{n_x}]^T$$

- ▶ weight matrix:

$$\mathbf{W} \in \mathbb{R}^{n_k \times n_x}$$

- ▶ bias vector:

$$\mathbf{b} = [b_1, \dots, b_{n_k}]^T$$

- ▶ activation vector hidden layer:

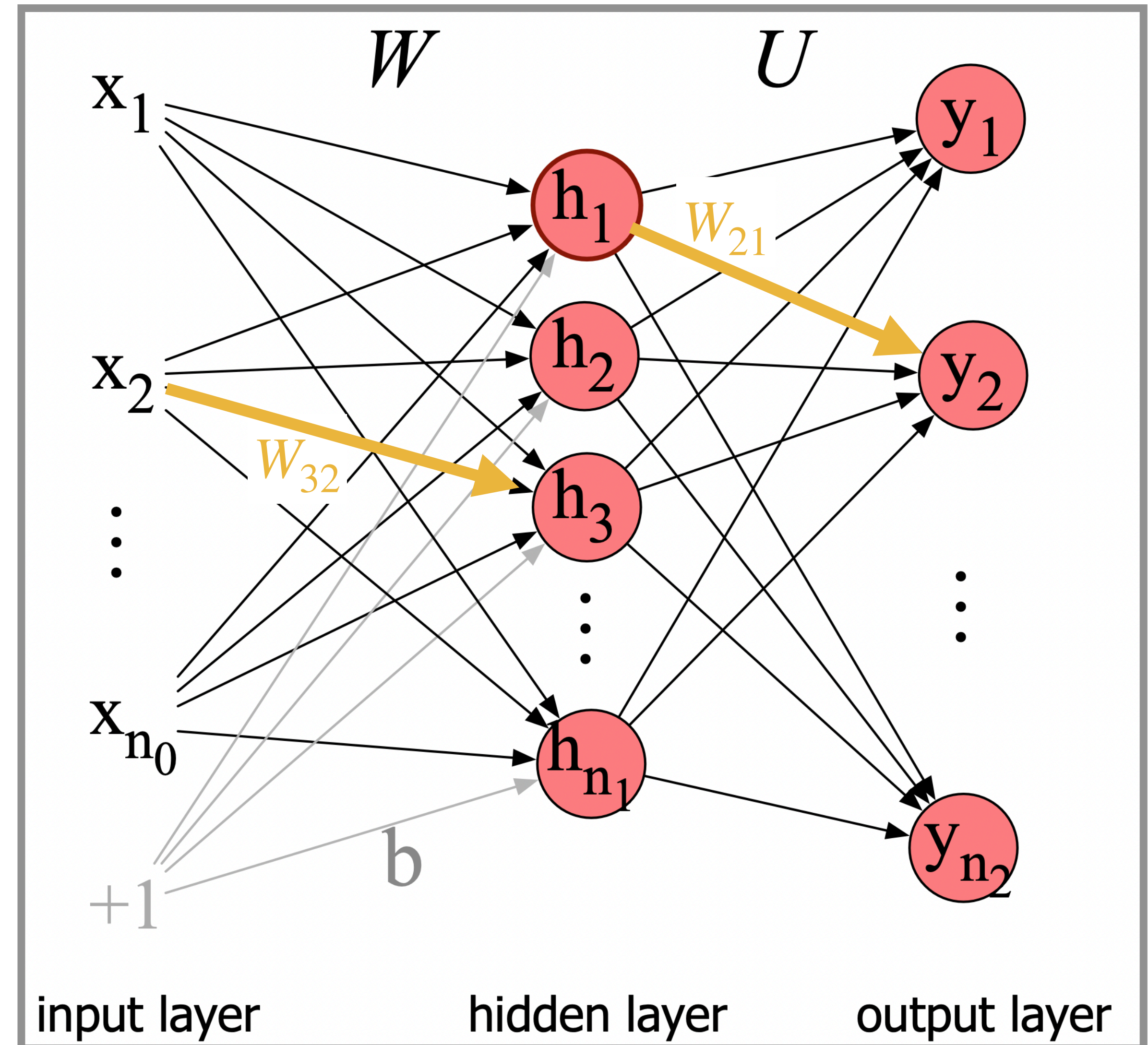
$$\mathbf{h} = f(\mathbf{W}\mathbf{x} + \mathbf{b}), \text{ with } f \in \{\sigma, \tanh, \dots\}$$

- ▶ weight matrix:

$$\mathbf{U} \in \mathbb{R}^{n_y \times n_k}$$

- ▶ prediction vector:

$$\mathbf{y} = g(\mathbf{U}\mathbf{h}), \text{ with } g \in \{\sigma, \text{soft-max}, \dots\}$$



Feedforward neural network (n hidden layer)

- ▶ anchoring in input:

$$\mathbf{a}^{[0]} = \mathbf{x} = [x_1, \dots, x_{n_x}]^T$$

- ▶ activation at layer n :

$$\mathbf{a}^{[n]} = f^{[n]}(\mathbf{W}^{[n]}\mathbf{a}^{[n-1]} + \mathbf{b}^{[n]})$$

with $f^{[n]} \in \{\sigma, \tanh, \dots\}$ if n is a hidden layer, or

with $f^{[n]} \in \{\sigma, \text{soft-max}, \dots\}$ if n is the output layer

