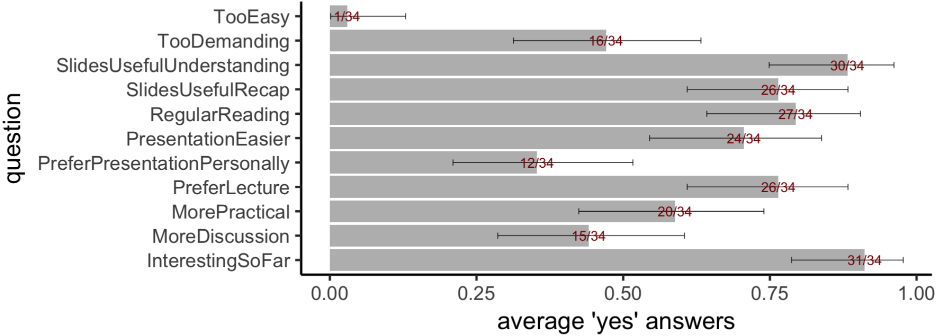
Models of Language Evolution

Replicator dynamic & signaling

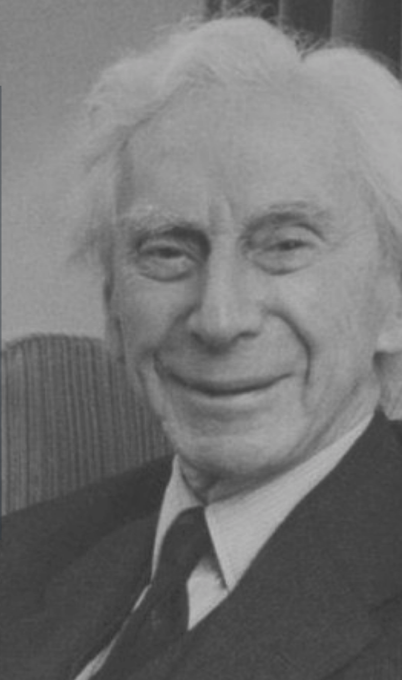
Michael Franke

Class survey results



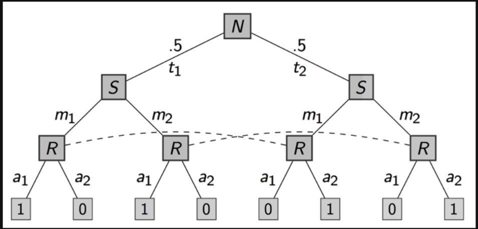
We can hardly suppose a parliament of hitherto speechless elders meeting together and agreeing to call a cow a cow and a wolf a wolf. The association of words with their meanings must have grown up ***by some natural process****, though at present the nature of the process is unknown.*

Bertrand Russell (1921) *The Analysis of Mind* p.190



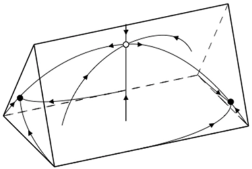
MEANING AS CONVENTION

equilibria of **signaling games**



David Lewis (1969) *Convention*

SIGNALING THEORY



evolutionary dynamics instead of equilibria

fitness-based selection **OR**
agent-level learning

meaning as **information content**

Brian Skyrms (2010) *Signals: Evolution, Learning, and Information*

Topics for today

1. replicator dynamic
2. evolutionary dynamics of signaling games

Replicator dynamic (discrete)

Replicator dynamic (continuous)

Dynamics of signaling

Levels of analysis in EGT

Static Solutions

Nash equilibrium
evolutionary stability
⋮

← are attractors of the dynamics static solutions?

Marco-Dynamics (Population-Level)

replicator dynamic
best response dynamic
⋮

← is macro-dynamics the *mean field* of the micro-dynamics?

Micro-Dynamics (Agent-Based)

imitate the best
conditional imitation
reinforcement learning
⋮

Replicator dynamic

- arguably, the most general formalization of fitness-proportional growth
- mathematically convenient:
  - discrete version $\leadsto$ numeric simulation
  - continuous version $\leadsto$ mathematical proof
- uniform formalism, multiply interpretable
- clear connection with stability & equilibrium notions

Utility in Mean-Field Populations

Recap

- let n_i be the number of agents playing action a_i
- let n be the size of the population
- population aggregate is a probability vector $\vec{p}$ where:

$$p_i = \frac{n_i}{n}$$

- if population is huge, the average payoff of a_i is:

$$\mathrm{U}(a_\mathrm{i}, \vec{p}) = \sum_j p_j \times \mathrm{U}(a_\mathrm{i}, a_\mathrm{j})$$

- $\mathrm{U}(a_\mathrm{i}, \vec{p})$ is the fitness of a_i (given the population state)
- $\mathrm{U}(\vec{p}) = \sum_i p_i \times \mathrm{U}(a_\mathrm{i}, \vec{p})$ is the average fitness in the population

Discrete-time replicator dynamics: Derivation

Assumptions

Average offspring of an individual playing a_i is a positive scaling function F of i's fitness $U(a_i, \vec{p})$: $F(x) = kx$ with $k > 0$.

- n'_i is the number of individuals playing a_i at the next discrete time step
- $n'_i = n_i \, F(U(a_i, \vec{p}))$

$$
\begin{aligned}
p'_i &= \frac{n'_i}{\sum_j n'_j} = \frac{n_i \, F(U(a_i, \vec{p}))}{\sum_j n_j \, F(U(a_j, \vec{p}))} \\
&= \frac{n_i \, k \, U(a_i, \vec{p})}{\sum_j n_j \, k \, U(a_j, \vec{p})} = \frac{n_i \, U(a_i, \vec{p})}{\sum_j n_j \, U(a_j, \vec{p})} \\
&= \frac{n \, p_i \, U(a_i, \vec{p})}{\sum_j n \, p_j \, U(a_j, \vec{p})} = \frac{p_i \, U(a_i, \vec{p})}{\sum_j p_j \, U(a_j, \vec{p})} = \frac{p_i \, U(a_i, \vec{p})}{U(\vec{p})}
\end{aligned}
$$

Discrete time replicator dynamic

$$p_i' = p_i \frac{U(a_\mathrm{i}, \vec{p})}{\sum_j p_j \, U(a_\mathrm{j}, \vec{p})} = p_i \frac{U(a_\mathrm{i}, \vec{p})}{U(\vec{p})} = \text{proportion of } i \times \frac{\text{fitness of } i}{\text{average fitness}}$$

If $p_i \neq 0$, frequency p_i of players of type a_i ...

... increases when i's fitness is higher than average;

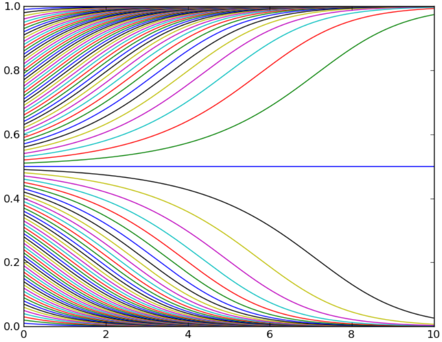
... decreases when i's fitness is lower than average;

... stays constant when i's fitness is exactly average.

If $p_i = 0$, then $p_i' = 0$.

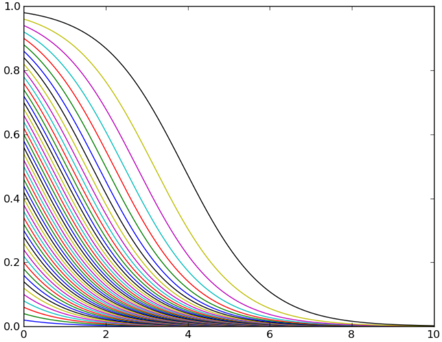
Time series

$$\text{Coordination: } U = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



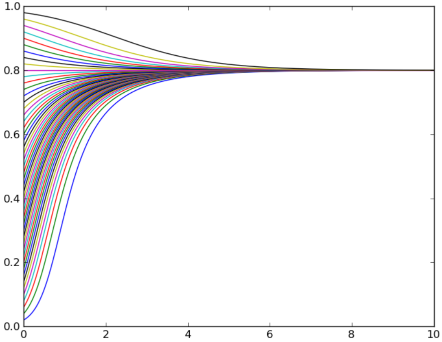
Time series

Prisoner's Dilemma: $U = \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}$



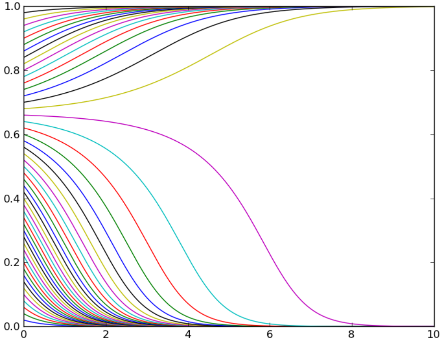
Time series

Hawks & Doves: $U = \begin{pmatrix} 1 & 7 \\ 2 & 3 \end{pmatrix}$



Time series

Coordination: $U = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$



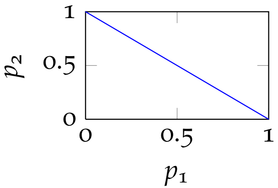
Source Code for Plots

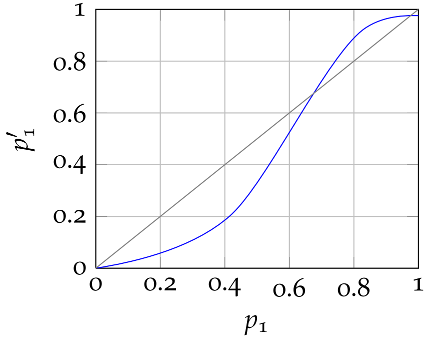
```
### plots the time series of the replicator dynamic for a 2 player symmetric game
### with 2 actions

# imports
from numpy import *
from pylab import *
from scipy.integrate import odeint

# utilities of the game
U = array([[2,0],[3,1]])

# starting configurations
# as proportions of first action
s_array = arange(0.02,1,0.02)

# time steps to obtain value for
t=arange(0,10,.01)

def expected_utility(p):
    return dot(U,array([p,1-p])) # careful: numpy uses the term "dot"-product here,
                                 # but it isn't't!

def overall_fitness(p):
    return dot(expected_utility(p),array([p,1-p]))

def replicator_dynamics(p,t):
    return array([p[0]*(expected_utility(p[0])[0] - overall_fitness(p[0]))])

for s in s_array:
    traj = odeint(replicator_dynamics,s,t)
    plot(t, traj)

show()

```

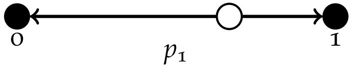
Analyzing the replicator dynamics

$$U = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

$$p_1' = \frac{p_1^2}{2(1-p_1)^2 + p_1^2}$$







Replicator dynamic (discrete)

Replicator dynamic (continuous)

Dynamics of signaling

Continuous time replicator dynamics

Derivation

$$\dot{p}_i = \frac{dp_i(t)}{dt} = \lim_{\delta t \to 0} \left[\frac{p_i(t+\delta t) - p_i(t)}{\delta t} \right]$$ [def. of derivative]

$$= p'_i - p_i$$ [discrete time RD gives limit step]

$$= p_i \, \frac{U(a_i, \vec{p})}{U(\vec{p})} - p_i$$ [def. of discrete time RD]

$$= p_i \, \frac{U(a_i, \vec{p}) - U(\vec{p})}{U(\vec{p})}$$ ["payoff-adjusted RD"]

$$\text{"="} \quad p_i \, [U(a_i, \vec{p}) - U(\vec{p})]$$ [drop constant denominator]

Continuous time replicator dynamic

$$\dot{p}_i = p_i \; [U(a_{\text{i}}, \vec{p}) - U(\vec{p})] \quad = \text{proportion of } i \times [\text{fitness of } i - \text{average fitness}]$$

If $p_i \neq 0$, frequency p_i of players of type a_{i} . . .

. . . increases when i's fitness is higher than average;

. . . decreases when i's fitness is lower than average;

. . . stays constant when i's fitness is exactly average.

If $p_i = 0$, then $\dot{p}_i = 0$.

Conditional imitation

Assumptions

- "mean field population": huge and homogeneous
- every agent plays fixed strategy for long periods of time
- occasionally i considers to adopt j's strategy
- switching probability is proportional to how much better j's strategy is than i's

Revision protocol

A revision protocol gives the average propensity (non-normalized probability) of agent i switching to agent j's strategy:

$$\rho_{ij}^{\vec{p}} = p_j \left[\mathrm{U}(a_j, \vec{p}) - \mathrm{U}(a_i, \vec{p}) \right]_+$$

$$= \text{proportion of } j \times \text{fitness difference between } j \text{ and } i \text{ (if } j \text{ is fitter)}$$

(e.g. Helbing, 1996; Schlag, 1998)

Derivation of the replicator dynamic

$$
\begin{aligned}
\dot{p}_i &= \text{flow into } i \; - \text{flow out of } i \\
&= \sum_j p_j \rho_{ji}^{\vec{p}} - \sum_j p_i \rho_{ij}^{\vec{p}} \\
&= \sum_j p_j p_i \left[\mathrm{U}(a_\mathrm{i}, \vec{p}) - \mathrm{U}(a_\mathrm{j}, \vec{p})\right]_+ - \sum_j p_i p_j \left[\mathrm{U}(a_\mathrm{j}, \vec{p}) - \mathrm{U}(a_\mathrm{i}, \vec{p})\right]_+ \\
&= p_i \sum_j p_j \left(\left[\mathrm{U}(a_\mathrm{i}, \vec{p}) - \mathrm{U}(a_\mathrm{j}, \vec{p})\right]_+ - \left[\mathrm{U}(a_\mathrm{j}, \vec{p}) - \mathrm{U}(a_\mathrm{i}, \vec{p})\right]_+ \right) \\
&= p_i \sum_j p_j \left(\mathrm{U}(a_\mathrm{i}, \vec{p}) - \mathrm{U}(a_\mathrm{j}, \vec{p})\right) \\
&= p_i \left(\sum_j p_j \, \mathrm{U}(a_\mathrm{i}, \vec{p}) - \sum_j p_j \, \mathrm{U}(a_\mathrm{j}, \vec{p}) \right) \\
&= p_i \left[\mathrm{U}(a_\mathrm{i}, \vec{p}) - \mathrm{U}(\vec{p})\right]
\end{aligned}
$$

Rest points, dynamic stability & attraction

- A **rest point** is a state $\vec{p}$ with $\dot{p}_i = 0$ for all i.
- a rest point $\vec{p}$ is **(weakly / Lyapunov) stable** iff:
    - all nearby points stay nearby
    - for all open neighborhoods U of $\vec{p}$ there is a neighborhood $O \subseteq U$ of $\vec{p}$ such that any point in O never migrates out of U
- a rest point $\vec{p}$ is **attractive** iff:
    - all nearby points converge to it
    - there is an open neighborhood U of $\vec{p}$ such that all points in U converge to $\vec{p}$
- **basin of attraction** of an attractive rest point:
    - biggest U with the above property
- a rest point $\vec{p}$ is **asymptotically stable** (aka. an **attractor**) iff:
    - all nearby points converge to it (on a path that stays close)
    - it is stable and attractive

Replicator dynamic, equilibrium & evolutionary stability

Equilibrium

1. NEs $\subseteq$ rest points
2. SNEs $\subseteq$ attractors
3. if an interior orbit converges to $\vec{p}$, then $\vec{p}$ is a NE
4. if a rest point is stable, then it is a NE

Evolutionary stability

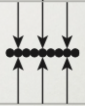
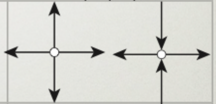
1. ESSs $\subseteq$ attractors
2. NSSs $\subseteq$ Lyapunov stable
3. all interior ESSs are *global* attractors, i.e., attract *all* interior points

Special case: "potential games" ($U = U^T$)

1. ESSs = attractors
2. every interior orbit converges (to a NE)

(e.g. Hofbauer and Sigmund, 1998)

How do we figure out the behavior at a point?

ESS	Asymptotically stable (it's a sink)	
NSS	Lyapunov stable, not necessarily a sink	
Symmetric Nash equilibria	Rest points, but not necessarily stable	
States that aren't symmetric Nash equilibria	May be a rest point (if it's a monomorphic population state or an equilibrium of a restricted game with extinct strategies), but cannot attract any points on the state space's interior	

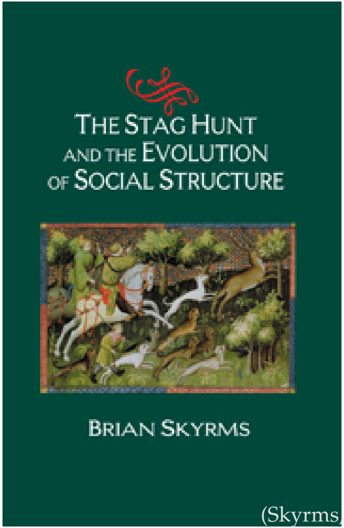
Replicator dynamic (discrete)

Replicator dynamic (continuous)

Dynamics of signaling

Early simulation evidence

In 2-2-2 Lewis games (with equiprobable states), all simulation runs of the (discrete, symmetric) RD converged to signaling systems.



(Skyrms, 1996)

Postitive result

2-2-2 Lewis game, equiprobable

In a 2-2-2 Lewis game with equiprobable states, the set of initial population states that do not converge to a signaling system under the replicator dynamics has Lebesgue measure zero.

Lebesgue measure zero: has an extension that does not stretch across all dimensions.

(Huttegger, 2007)

Negative result

2-2-2 Lewis game, non-equiprobable

In a 2-2-2 Lewis game with non-equiprobable states, the set of initial population states that do not converge to a signaling system under the replicator dynamics has positive Lebesgue measure.

Reason: there are now mixed NSSs; these must be attractors, because they are Lyapunov stable (generally) and interior points must converge to a NEs (partnership games).

(Huttegger, 2007)

Negative result

n-*n*-*n* Lewis games, equiprobable

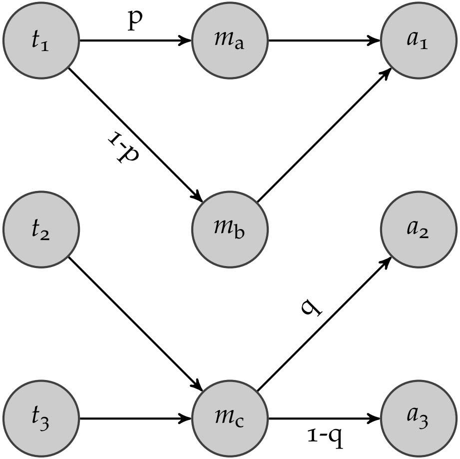
In a *n*-*n*-*n* Lewis game with equiprobable states, the set of initial population states that do not converge to a signaling system under the replicator dynamics has positive Lebesgue measure.

Reason: there are now mixed neutrally stable strategies (NSSs, so-called **partial pooling equilibria**).

Basin of attraction: ca. 5% of simulation runs in the symmetric RD converge to partial pooling equilibria.

(Huttegger, 2007; Pawlowitsch, 2008)

Party pooper: partial pooling



(Pawlowitsch, 2008)

Positive result

3-3-3 Lewis game, equiprobable

In a 3-3-3 Lewis game with equiprobable states, the set of initial population states that do not converge to a signaling system under the replicator-mutator dynamics (with uniform small mutation rates) seems to have Lebesgue measure zero.

Replicator-mutator dynamics: replicator dynamics with mutation.

(Huttegger et al., 2010)

Upshot

While the evolution of perfect information transfer is not an evolutionary certainty (even in idealized models), at least partial information transfer seems almost guaranteed by success-conditioned selection of communicative strategies.

Homework

read this paper:

- Simon Kirby et al. (2014). "Iterated Learning and the Evolution of Language". In: *Current Opinion in Neurobiology* 28, pp. 108–114

References

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