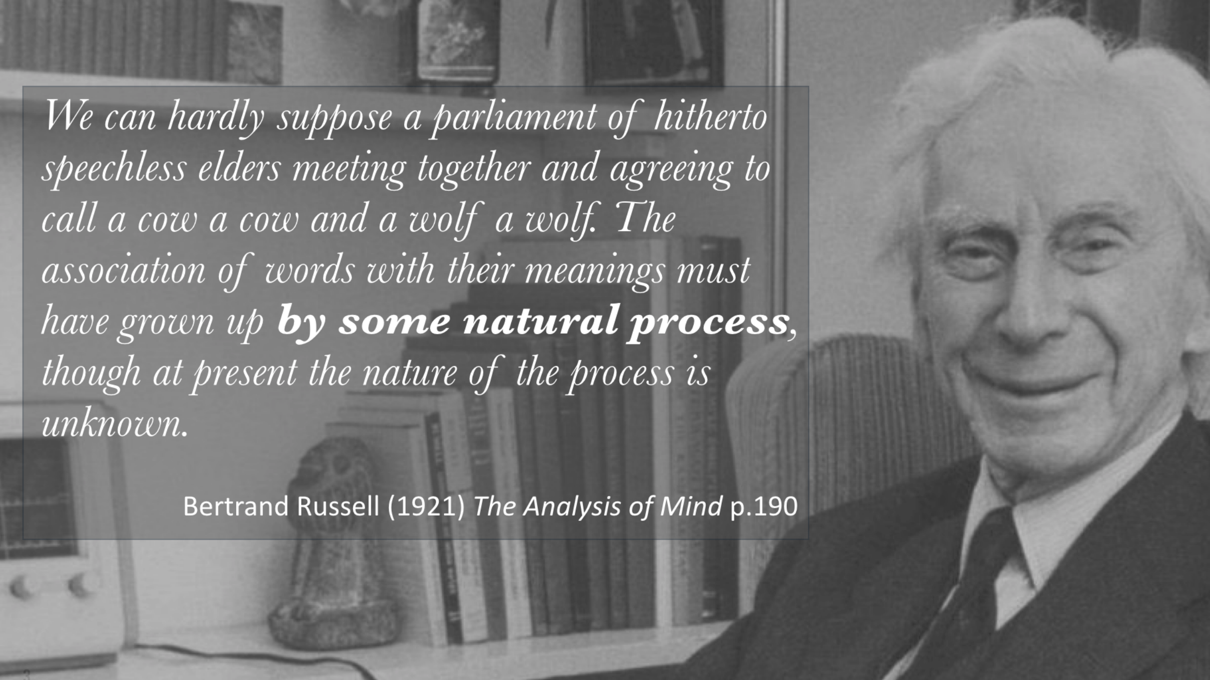
Models of Language Evolution

Evolutionary game theory & signaling games

Michael Franke

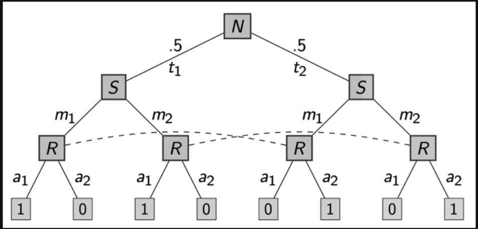


We can hardly suppose a parliament of hitherto speechless elders meeting together and agreeing to call a cow a cow and a wolf a wolf. The association of words with their meanings must have grown up ***by some natural process****, though at present the nature of the process is unknown.*

Bertrand Russell (1921) *The Analysis of Mind* p.190

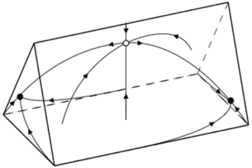
MEANING AS CONVENTION

equilibria of **signaling games**



David Lewis (1969) *Convention*

SIGNALING THEORY



evolutionary dynamics instead of equilibria

fitness-based selection **OR**
agent-level learning

meaning as **information content**

Brian Skyrms (2010) *Signals: Evolution, Learning, and Information*

Topics for today

1. (flavors of) game theory
2. signaling games (& conversion into symmetric form)
3. Nash equilibrium (in symmetric games)
4. evolutionary stability
5. meaning of signals

Game Theory

Signaling games

Population Games

(Rational) Choice Theory

Decision Theory: a single agent's solitary decision

Game Theory: multiple agents' interactive decision making

Game Theory

- abstract mathematical tools for modeling and analyzing multi-agent interaction
- since 1940: **classical game theory** (von Neumann and Morgenstern)
  - *perfectly rational agents* ::: Nash equilibrium
  - initially promised to be a unifying formal foundation for all social sciences
  - Nobel laureates: Nash, Harsanyi & Selten (1994), Aumann & Schelling (2006)
- since 1970: **evolutionary game theory** (Maynard-Smith, Prize)
  - *boundedly-rational agents* ::: evolutionary stability & replicator dynamics
  - first applications in biology, later also elsewhere (linguistics, philosophy)
- since 1990: **behavioral game theory** (Selten, Camerer)
  - studies interactive decision making in the lab
- since 1990: **epistemic game theory** (Harsanyi, Aumann)
  - studies which (rational) beliefs of agents support which solution concepts

Games vs. Behavior

Game: abstract model of a recurring interactive decision situation

- think: a model of the environment

Strategies: all possible ways of playing the game

- think: a full contingency plan or a (biological) predisposition for how to act in every possible situation in the game

Solution: subset of "good strategies" for a given game

- think: strategies that are in equilibrium, rational, evolutionarily stable, the outcome of some underlying agent-based optimization process etc.

Solution concept: a general mapping from any game to its specific solution

- examples: Nash equilibrium, evolutionary stability, rationalizability etc.

Kinds of Games

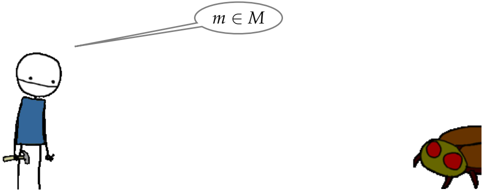
uncertainty	choice points	
	simultaneous	in sequence
no	*strategic / static*	*dynamic / sequential* with *complete* info
yes	*Bayesian*	*dynamic / sequential* with *incomplete* info

Game Theory

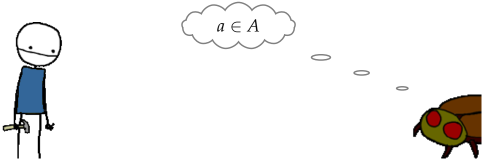
Signaling games

Population Games

sender knows state, but receiver does not



sender sends a signal



receiver chooses act

State-Act Payoff Matrix

	a_1	a_2	$\dots$
t_1	1,1	0,0	
t_2	1,0	0,1	
$\vdots$			

(Lewis, 1969)

Signaling game

A **signaling game** is a tuple

$$\langle \{S, R\}, T, \Pr, M, A, \mathrm{U}_S, \mathrm{U}_R \rangle$$

with:

- $\{S, R\}$ set of **players**
- T set of **states**
- $\Pr$ **prior beliefs**: $\Pr \in \Delta(T)$
- M set of **messages**
- A set of receiver **actions**
- $\mathrm{U}_{S,R}$ **utility functions**: $T \times M \times A \to \mathbb{R}$.

Talk is **cheap** iff for all t, m, m', a and $X \in \{S, R\}$:

$$\mathrm{U}_X(t, m, a) = \mathrm{U}_X(t, m', a)\,.$$

Otherwise we speak of **costly signaling**.

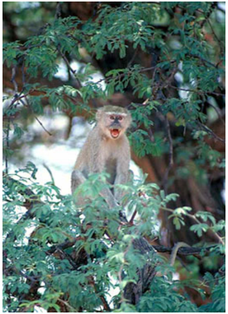
model of the context/environment/world

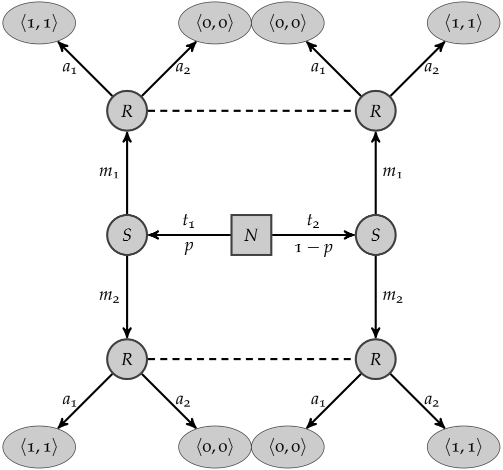
Example (2-2-2 Lewis game)

2 states, 2 messages, 2 acts

	$\Pr(t)$	a_1	a_2
t_1	p	1, 1	0, 0
t_2	$1-p$	0, 0	1, 1

Example (Alarm calls)





Strategies

Pure

$s \in M^T$ $r \in A^M$ fixed contingency plan

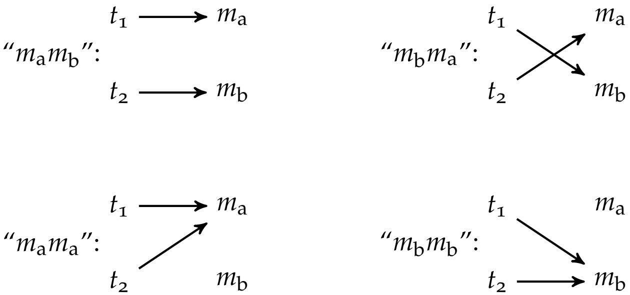
Mixed

$\tilde{s} \in \Delta(M^T)$ $\tilde{r} \in \Delta(A^M)$ uncertainty about plan

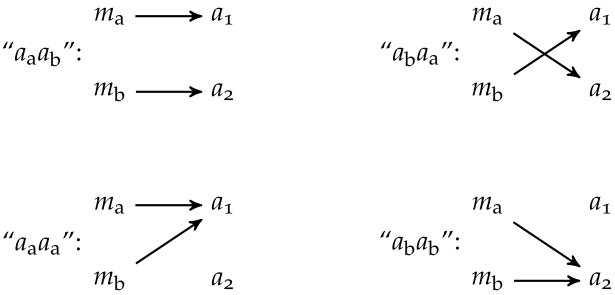
Behavioral

$\sigma \in (\Delta(M))^T$ $\rho \in (\Delta(A))^M$ probabilistic plan

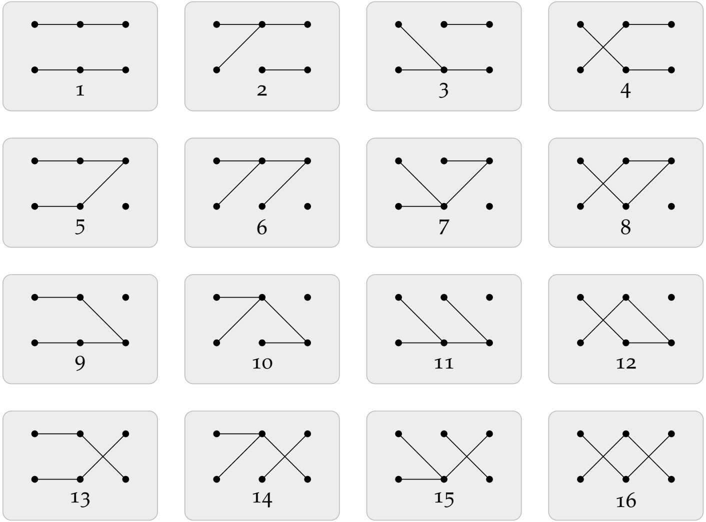
Pure sender strategies in the 2-2-2 Lewis game



Pure receiver strategies in the 2-2-2 Lewis game



All pairs of sender-receiver pure strategies for the 2-2-2 Lewis game



Game Theory

Signaling games

Population Games

(One-Population) Symmetric Game

A (one-population) symmetric game is a pair $\langle A, U \rangle$, where:

- A is a set of acts, and
- $U : A \times A \to \mathbb{R}$ is a utility function (matrix).

Example (Prisoner's dilemma)

$$U = \begin{array}{c} \\ a_c \\ a_d \end{array} \begin{array}{c} \begin{array}{cc} a_c & a_d \end{array} \\ \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix} \end{array}$$

Example (Hawk & Dove)

$$U = \begin{array}{c} \\ a_h \\ a_d \end{array} \begin{array}{c} \begin{array}{cc} a_h & a_d \end{array} \\ \begin{pmatrix} 1 & 7 \\ 2 & 3 \end{pmatrix} \end{array}$$

Mixed strategies in symmetric games

A mixed strategy in a symmetric game is a probability distribution $\sigma \in \Delta(A)$.

Utility of mixed strategies defined as usual:

$$\mathrm{U}(\sigma, \sigma') = \sum_{a,a' \in A} \sigma(a) \times \sigma(a') \times \mathrm{U}(a, a')$$

Nash Equilibrium in Symmetric Games

A mixed strategy $\sigma \in \Delta(A)$ is a **symmetric Nash equilibrium** iff for all other possible strategies σ':

$$U(\sigma, \sigma) \geq U(\sigma', \sigma).$$

It is **strict** if the inequality is strict for all $\sigma' \neq \sigma$.

Examples

Prisoner's Dilemma

$$U = \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}$$

symmetric NE: $\langle 0, 1 \rangle$

Hawk & Dove

$$U = \begin{pmatrix} 1 & 7 \\ 2 & 3 \end{pmatrix}$$

symmetric NE: $\langle .8, .2 \rangle$

Symmetrizing asymmetric games

Example: signaling game

- big population of agents
- every agent might be sender or receiver
- an agent's strategy is a pair $\langle s, r \rangle$ of pure sender and receiver strategies
- utilities are defined as the average of sender and receiver role:

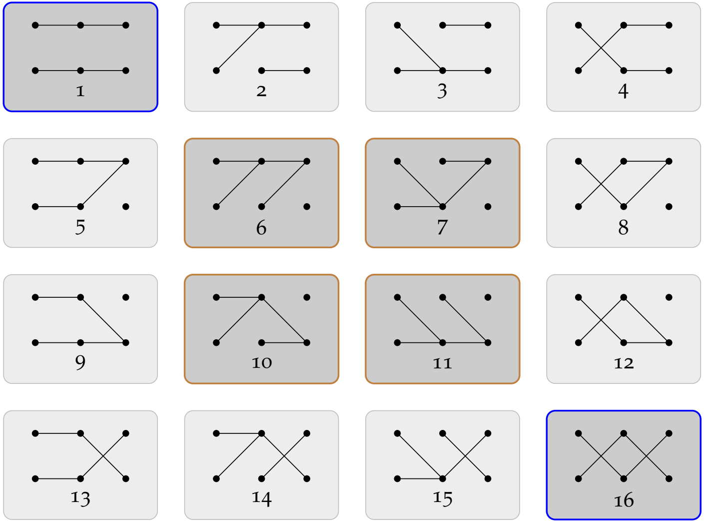
$$U(\langle s, r \rangle, \langle s', r' \rangle) = {}^1\!/_2(U_S(s, r') + U_R(s', r))$$

Example (Symmetrized 2-2-2 Lewis game)

		s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}	s_{16}
s_1	$\langle m_1, m_1, a_1, a_1 \rangle$	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5
s_2	$\langle m_1, m_1, a_1, a_2 \rangle$	.5	.5	.5	.5	.75	.75	.75	.75	.25	.25	.25	.25	.5	.5	.5	.5
s_3	$\langle m_1, m_1, a_2, a_1 \rangle$	.5	.5	.5	.5	.25	.25	.25	.25	.75	.75	.75	.75	.5	.5	.5	.5
s_4	$\langle m_1, m_1, a_2, a_2 \rangle$	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5
s_5	$\langle m_1, m_2, a_1, a_1 \rangle$	.5	.75	.25	.5	.5	.75	.25	.5	.5	.75	.25	.5	.5	.75	.25	.5
s_6	$\langle m_1, m_2, a_1, a_2 \rangle$	.5	.75	.25	.5	.75	1	.5	.75	.25	.5	0	.25	.5	.75	.25	.5
s_7	$\langle m_1, m_2, a_2, a_1 \rangle$	.5	.75	.25	.5	.25	.5	0	.25	.75	1	.5	.75	.5	.75	.25	.5
s_8	$\langle m_1, m_2, a_2, a_2 \rangle$	.5	.75	.25	.5	.5	.75	.25	.5	.5	.75	.25	.5	.5	.75	.25	.5
s_9	$\langle m_2, m_1, a_1, a_1 \rangle$	.5	.25	.75	.5	.5	.25	.75	.5	.5	.25	.75	.5	.5	.25	.75	.5
s_{10}	$\langle m_2, m_1, a_1, a_2 \rangle$	.5	.25	.75	.5	.75	.5	1	.75	.25	0	.5	.25	.5	.25	.75	.5
s_{11}	$\langle m_2, m_1, a_2, a_1 \rangle$	.5	.25	.75	.5	.25	0	.5	.25	.75	.5	1	.75	.5	.25	.75	.5
s_{12}	$\langle m_2, m_1, a_2, a_2 \rangle$	.5	.25	.75	.5	.5	.25	.75	.5	.5	.25	.75	.5	.5	.25	.75	.5
s_{13}	$\langle m_2, m_2, a_1, a_1 \rangle$	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5
s_{14}	$\langle m_2, m_2, a_1, a_2 \rangle$	.5	.5	.5	.5	.75	.75	.75	.75	.25	.25	.25	.5	.5	.5	.5	.5
s_{15}	$\langle m_2, m_2, a_2, a_1 \rangle$	.5	.5	.5	.5	.25	.25	.25	.25	.75	.75	.75	.75	.5	.5	.5	.5
s_{16}	$\langle m_2, m_2, a_2, a_2 \rangle$	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5

non-strict symmetric NE, strict symmetric NE

All pairs of sender-receiver pure strategies for the 2-2-2 Lewis game



Reading for Next Class

Brian Skyrms (2010) “Information” Chapter 3 of “Signals” OUP.

References

Lewis, David (1969). *Convention. A Philosophical Study*. Cambridge, MA: Harvard University Press.